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FORMAL LOGIC

Formal logic is the *a priori* study of statements called propositions and of deductive arguments by way of identifying structures or logical forms in these elements and expressing them in symbolic notation in order to test their validity. Therefore, formal logic is not empirical study because it does not depend on *a posteriori* observations for data.

DEDUCTIVE ARGUMENT

A deductive argument is one in which the conclusion, a proposition, follows necessarily from the premises, another proposition or set of propositions such that denying the conclusion would be inconsistent or contradictory.

CONDITIONS OF PROOF FOR A SOUND ARGUMENT

In order to prove the truth of the conclusion of a deductive argument,

- the premises must be true;
- the deduction must be logically correct.

If a deductive argument meets these conditions, it is called sound.

Whilst formal logic will deal with the first condition, it cannot determine the truth or falsity of the premises where the propositions are *a posteriori*, contingent or synthetic. The truth or falsity of the premises, in this case, rests with the empiricist.

Therefore in proving the soundness of the deductive argument for the existence of God I shall use both formal logic to prove the deduction and both innate and empirical evidence to prove the truth of the premises from which the existence of God as a conclusion necessarily follows.

If, however, only the first condition, that the conclusion is logically deducible from its premises, the argument is said to be deductively valid even though the premises are false or not known to be true. However, the argument would be unsound.

For example, the argument that:

- Every dog is a mammal.
- Some quadrupeds are dogs.
- Therefore, some quadrupeds are mammals.

is valid, because they can expressed in the same valid logical inference form:

- Every *X* is a *Y*.
- Some *Z's* are *X's*.
- Therefore, some *Z's* are *Y's*.

However, the soundness of each argument depends also on whether the premises are true or false, and this is outside the scope of formal logic if the propositions *a posteriori*, contingent or synthetic.

FORMAL LOGIC

VALID INFERENCE FORMS

The letters—*X*, *Y*, and *Z*—are called variables, like the *x* in algebra, and by which an inference form is produced by uniformly replacing all the variables in it with expressions that make sense in the context.

Every instance of the inference form will be logically valid if any variable that makes the premises true also ensures the truth of the conclusion. In other words, a valid inference form is one for which no instance of it can have true premises and a false conclusion.

In contrast, the following inference form is not valid:

- Every *X* is a *Y*.
- Some *Z's* are *Y's*.
- Therefore, some *Z's* are *X's*.

because there are instances in which the premises are true but the conclusion is false. For example,

- Every dog is a mammal.
- Some winged creatures are mammals.
- Therefore, some winged creatures are dogs.

Formal logic identifies and validates inference forms and derives the relations that hold among valid ones.

VALID PROPOSITION FORMS

A proposition form is a combination of propositions and it is logically valid if it is true for all instances of the propositions, for example,

- Everything is *X* or not *X*

Formal logic involves proposition forms and inference forms.

SYSTEM OF LOGIC

A system of logic is made up of:

- a symbolic apparatus comprising a set of symbols, the rules for combination of symbols into formulae and the rules for manipulation of formulae
- definitions of these symbols and formulas

A system of logic without symbolic and formulaic definitions is called uninterpreted or purely formal.

A system of logic with symbolic and formulaic definitions is called interpreted.

An axiomatic system of logic is one which is based on unproved formulas called axioms from which further formulas called theorems are proved. The proof of a theorem depends solely on which formulas are taken as axioms and the rules for deriving theorems from axioms, and not on the meaning of the axioms or theorem.

PROPOSITIONAL CALCULUS

Propositional calculus is the simplest and most basic branch of logic because the propositions are unanalysed.

The symbols used in propositional calculus are:

- variables
- operators
- brackets

VARIABLES

Variables, such as *p*, *q* and *r* represent unspecified propositions in formulas into which only sentences may be inserted.

OPERATORS

Operators create a new proposition from one or more given propositions which are called the arguments of the operator.

The operators are:

- ~, which means "not"
- ·, which means "and"
- ∨, which means "or"
- ⊃, which means "if…then"
- ≡, which means "is equivalent to"

BRACKETS

Brackets make it possible to distinguish, for example, between

$$p \cdot (q \vee r)$$

and

$$(p \cdot q) \vee r$$

In other words, the first proposition is "both *p* and either *q* or *r*", and this differs in meaning to the second proposition "either both *p* and *q* or *r*".

TRUTH-FUNCTIONAL AXIOMS AND OPERATORS

It is assumed:

- that every proposition is either true or false
- that no proposition is both true and false

Truth and falsity are called the truth values of propositions.

If the truth value of a proposition can be determined given the truth values of the arguments and the operator which forms the proposition, then the proposition is called the truth function of the operator's argument and the operator is called a truth functional operator.

The truth functionality of operators is set out below for any two propositions *p* and *q*:

- the truth function, "not *p*", is false when *p* is true, and true when *p* is false
- the truth function, "*p* and *q*", is true when *p* and *q* are both true, and as false in all other cases
- the truth function, "*p* or *q*", is false when *p* and *q* are both false, and true in all other cases
- the truth function, "if *p* then *q*", is false when *p* is true and *q* is false, and true in all other cases
- the truth function, "*p* is equivalent to *q*" is true when *p* and *q* are both true or when both are false, and false when *p* and *q* have different truth values

and also in the table below, where,

- true is "T";
- false is "F";

all possible combinations of truth values of the operators' arguments are listed to the left of the vertical line.

p	$\sim p$	p	q	$p \cdot q$	$p \vee q$	$p \supset q$	$p \equiv q$
T	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T
		F	T	F	T	F	F
		F	F	F	F	T	T

LAWS OF THOUGHT

The laws of thought are the three fundamental laws of logic, and these are:

- the law of non-contradiction
- the law of excluded middle
- the principle of identity

LAW OF NON-CONTRADICTION

For all propositions p, it is impossible for both p and not p to be true, or

$$\sim(p \cdot \sim p)$$

LAW OF EXCLUDED MIDDLE

Either p or not p must be true, or

$$p \vee \sim p$$

There is no third or middle true proposition between them.

PRINCIPLE OF IDENTITY

A thing x is identical with itself, or

$$x \equiv x$$

CRITICISMS OF THE LAWS OF THOUGHT

L.E.J. Brouwer, a Dutch mathematical intuitionist, rejected the law of excluded middle in mathematical proofs that used infinities. Since an actual infinity does not exist in the real world and the argument for the existence of God relates to the real world, the law of excluded middle is valid for my purpose.

Jan Jukasiewicz of the Polish school of logic developed a propositional calculus that had a third truth-value of neither truth nor falsity to deal with future contingent events when the laws of non-contradiction and excluded middle both fail. Since the laws of non-contradiction and excluded middle will be applied to past events and not future contingent events, they remain valid for my purpose.